Price Discrimination in Oligopolies with Best-Response Asymmetry

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January 2005

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Abstract

Corts (1998) argues that in oligopolies with best-response asymmetry (i.e. if the strong market of one firm is the weak market of the other firm), price discrimination may result both in an decrease or in an increase of all prices. It is, however, an open theoretical question when such situations indeed occur. The present paper analyses conditions on demand functions ruling out the possibility that price discrimination causes all prices to move in the same direction, but it also discusses examples when they do.

Keywords: price discrimination, oligopoly, best-response asymmetry

JEL-classification: D43, L13

1 Introduction

Price discrimination is most often analysed in the context of monopolies. But all firms that have some scope to set prices have an incentive to price discriminate if they act on submarkets with different demand structures. If possible, oligopolistic firms will set lower prices on submarktets with a high price elasticity of demand[1]. In his analysis of price discrimination in oligopolies, Corts (1998) points out that for the case of differentiated products, firms need not rank the submarkets in the same way. Two basic cases can be distinguished. A duopoly model exhibits *best response symmetry* if both firms rank the same consumer group as the strong market with a low price elasticity of demand for their product variety. In contrast, *best response asymmetry* means that the strong market of one firm is the weak market of the other firm.

Examples of best response symmetry include discounts to students or senior citizens, and different prices for pharmaceuticals in low income and high income countries. Best-response asymmetry occurs, if firms in a sense have a “home market”. Examples referred to in the literature include supermarkets with different locations which have an incentive to give discounts (i.e. coupons) to consumers living close to other supermarkets (Bester and Petrakis 1996) and electricity suppliers if consumers have switching costs of changing their supplier (Schulz 1999). These models have the common feature that *all* consumers will buy the product variety of the home market they belong to (i.e. go shopping at the nearest supermarket or remain with their electricity supplier) as long as its price does not exceed the rival’s price. As a consequence, demand curves for the individual submarkets in general have a kink. In contrast, Corts (1998) assumes differentiable profit functions. A suitable example for best-response asymmetry in this case is international price discrimination on markets with intra-industry trade and a home bias in demand, that reflects a certain preference for domestic products (Feuerstein 2003).

International price discrimination may thus take the form of both best-response symmetry or best-response asymmetry. An illustrative example is the European car market

[1] Early papers on price discrimination in oligopolies include Neven and Phlips (1985) and Holmes (1989).

	Italy	France
Lancia Lybra	19800	18600
Peugeot 806	18900	20100

Table 1: Car prices May 2001 in euro (Source: European Commission 2001)

where substantial price differences, that can amount to several thousand euro for an identical model, are observed[2]. Empirical studies have shown, that there are both country effects and a home bias effect consisting of an additional mark-up for domestically produced cars (Mertens/Ginsburgh 1985, Verboven 1996, Lutz 2003). Table 1 shows an example for the home bias effect. The Lancia Lybra is more expensive in Italy than in France, whereas Peugeot sells its model 806 at a higher price in France than in Italy.

In case of best-response asymmetry, the effects of price discrimination may differ substantially from the results for monopolies. The uniform price of a monopolist always lies between his discriminating prices and the same result holds for oligopolists acting on markets with best-response symmetry, but it need not hold in case of best-response asymmetry. According to Corts (1998), it is both possible that price discrimination results in unequivocally higher or in unequivocally lower prices than if each firm must set a uniform price. The result that permitting price discrimination in an oligopolistic setting may *lower* prices for all consumers is also contained in Bester and Petrakis (1996) and Schulz (1999). However, in these papers the effect is based on the kink of the demand functions as described above. If differentiable demand functions are considered, it is an open problem for which demand functions price discrimination causes all prices to move in the same direction. Moreover, as is shown in the present paper,

[2]Within the single market, consumers may buy their car wherever they want, and they pay the taxes of their country of residence. However, the old EU block exemption for distribution system of cars, that expired in September 2002, allowed car producers to prevent commercial arbitrage and enabled them to essentially segment markets. The new block exemption is expected to enhance arbitrage and to lower price differences.

applying Corts' reasoning leads to demand systems with the peculiar feature that total demand rises in response to a price increase, limiting the relevance of his result.

In this paper, we analyse price discrimination in oligopolies with best-response asymmetry using the same type of model as in Corts (1998). We discuss properties of the demand functions that exclude the possibility that prices unambiguously rise (or fall) when price discrimination is introduced, but we also specify conditions when they do. In section 3, we show that Corts' proof is not valid any more if the plausible assumption is made that total demand in each country depends negatively on both prices. Section 4 assumes that the markets are symmetric in the sense of mirror images and gives conditions that ensure that the uniform prices will lie between the discriminating prices of the firms. In particular, symmetry and linearity of the demand functions are sufficient for this result. But we can also specify sufficient conditions for prices to move in the same direction by introducing asymmetry or non-linearity. Section 5 considers the case of linear demand functions with some asymmetry in the parameters that make it possible that one of the firms increase (decrease) both of its prices, implying that three of the four prices increase (decrease). Section 6 gives an example of a symmetric duopoly with best response asymmetry, in which the uniform price is higher than all prices in the discriminating equilibrium. Section 7 summarizes and assesses the results. In particular, the intuition of the effects of price discrimination is discussed.

2 The Model

We consider a duopoly model with firms A and B. Each firm produces one variety of the good considered and sells it on the two submarkets 1 and 2. Firms compete in prices, i.e. there is Bertrand competition in a differentiated product. Marginal production costs c are constant and identical for both firms. The price, the quantity and the profit of firm J in market i are denoted by p^{iJ}, x^{iJ} and π^{iJ}, with $i = 1, 2,\ J = A, B$. Demand functions (and equivalently profit functions) are twice continuously differentiable, and the derivatives with respect to prices are denoted by the corresponding subscripts $J = A, B$, e.g. $\pi^{iA}_A = \dfrac{d\pi^{iA}}{dp^{iA}}$. On each submarket, demand for each product variety $x^{iJ}(p^{iJ}, p^{iK})$, $J \neq K$, depends negatively on the own price and positively on the price of the other product variety, i.e. $x_J^{iJ} < 0$ and $x_K^{iJ} > 0$.

We make the following assumptions which are standard in models of price competition with differentiated products.

$$\pi_{JJ}^{iJ} < 0, \quad i = 1, 2\ , \ \ J = A, B\ , \tag{1}$$

$$\pi_{JK}^{iJ} > 0, \quad J \neq K, \tag{2}$$

$$\pi_{JJ}^{iJ} + \pi_{JK}^{iJ} < 0, \quad J \neq K. \tag{3}$$

Assumption (1) ensures that the firms' best response functions are well defined, assumption (2) implies that prices are strategic complements, and assumption (3) is a stability condition implying that the reaction curves $R^{iJ}(p^{iK}),\ J \neq K$, have a slope smaller than one and there is a unique price equilibrium in each country.

Moreover, it is assumed that each firm can unequivocally rank the two submarkets according to the price elasticity of demand for its product variety, i.e. that a firm's reaction curves for the two markets do not cross. The profit functions exhibit *best response symmetry* if for both firms the price elasticity of demand is relatively high (low) on the same market. In contrast, there is *best response asymmetry*, if the strong market of one firm is the weak market of the other firm. Without loss of generality, let market 1 be the strong market of firm A. In this case best response asymmetry means

$$R^{1A}(p) > R^{2A}(p) \qquad \text{and} \qquad R^{2B}(p) > R^{1B}(p). \tag{4}$$

Both cases are illustrated in figure 1.

Since an important example for best response asymmetry is intra-industry trade with a home bias in demand, we will often refer to the submarkets as *country* 1 and 2. Moreover, we can think of firm A being located in country 1 and firm B being located in country 2, meaning that for each firm, the strong market is the domestic market and the weak market is the export market. Of course, these names only serve as an illustration, and the results of the model refer to other interpretations as well.

Corts (1998) has shown that if no price discrimination is possible, the uniform price equilibrium lies in the region U that is defined as the inner points of the quadrangle

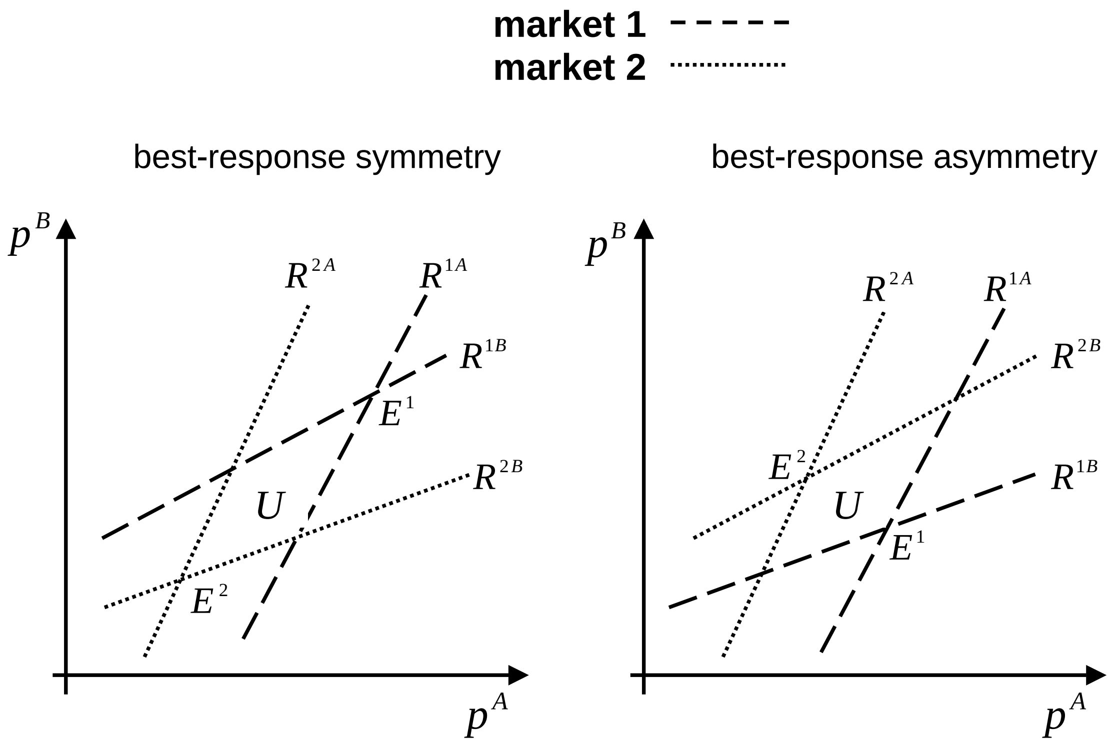

Figure 1: Price discrimination in oligopolies

generated by the four reaction curves. Whereas for best-response symmetry this implies that for each firm the uniform price lies between its two discriminating prices, Corts points out that with best-response asymmetry, this need not be the case. In addition he claims that every element of the region U is a uniform price equilibrium for some profit functions generating that region U. In case of best response asymmetry, this would imply that price discrimination may lead to lower prices for all consumers, and that there are also cases when it leads to higher prices for all consumers. The next section discusses Corts' reasoning and shows that it implies peculiar features of the demand functions, and is thus not convincing.

3 Generating equilibria by weighting demands

A firm that sells in both countries but can only set a uniform price has to weigh the effects of a price change on the profits earned on the two markets. A monopolist will choose a uniform price that lies between the profit-maximizing prices on the segmented markets. For an oligopolistic firm, an analogous argument implies that the reaction curve for uniform prices runs between the reaction curves for the individual countries.

Moreover, it is straightforward to see that each price between the two separate prices of a discriminating monopolist is a uniform price for some demand functions leading to the same discriminating prices. Assume for a moment, that there is only one firm facing demand functions x^1 and x^2. Let the strong market be country 1, i.e. $p^{1M} > p^{2M}$, and choose $\hat{p}$ such that $p^{2M} < \hat{p} < p^{2M}$. As $\pi_M^{1M}(\hat{p}) > 0$ and $\pi_M^{2M}(\hat{p}) < 0$, it is clear that there exist weights $\alpha^1 > 0$ and $\alpha^2 > 0$ such that $\alpha^1\pi_M^{1M}(\hat{p}) + \alpha^2\pi_M^{2M}(\hat{p}) = 0$, meaning that $\hat{p}$ is the monopolist's profit-maximizing uniform price for the demand functions $\hat{x}^1 = \alpha^1 x^1$ and $\hat{x}^2 = \alpha^2 x^2$. The weights α^1 and α^2 have a direct interpretation as a change in size of the respective markets.

Corts (1998, Proposition 2) applies this reasoning to oligopolies. For every pair of prices $\hat{p} = (\hat{p}^A, \hat{p}^B)$ contained in the region U there are weights α^{iJ}, $i = 1, 2$, $J = A, B$, such that

$$\alpha^{1J}\pi_J^{1J}(\hat{p}) + \alpha^{2J}\pi_J^{2J}(\hat{p}) = 0 \qquad J = A, B, \tag{5}$$

implying that $\hat{p}$ is a uniform price equilibrium for the demand functions $\hat{x}^{iJ} = \alpha^{iJ}x^{iJ}$, which result in the same region U. A large α^{iJ} shifts firm $J's$ uniform price reaction function towards its reaction function for country i and vice versa, and the weights can be chosen in a way that the uniform price reaction functions of both firms run through a given point $\hat{p} \in U$. However, the interpretation of the weights is not as simple as in case of a monopolist. A large weight α^{iJ} cannot be interpreted as a proportionate increase of the size of a country (or consumer group) any more, but refers to the country's demand for one product variety only, thereby changing the structure of demand in this country. In particular, for points close to the boundary of the region U, the reasoning leads to demand systems with the peculiar feature that total demand in a country depends *positively* on one of the prices. This result is formalized in the following proposition.

Proposition 1 Let profit functions $\pi^{iJ} = (p^{iJ} - c)x^{iJ}$, $i = 1, 2$, $J = A, B$ meet assumptions (1)-(3).

For every $\tilde{p}$ on the boundary of U, $\tilde{p} \neq E^1,\ E^2$, there is a neighborhood $V(\tilde{p})$ such that for every $p \in V(\tilde{p}) \cap U$ the following holds:

If

$$\alpha^{1A}\pi_A^{1A}(p) + \alpha^{2A}\pi_A^{2A}(p) = 0 \quad \text{and} \quad \alpha^{1B}\pi_B^{1B}(p) + \alpha^{2B}\pi_B^{2B}(p) = 0$$

for positive real numbers α^{iJ},
(i.e. if p is a uniform price equilibrium for the demand functions $\hat{x}^{iJ} = \alpha^{iJ}x^{iJ}$),

then for at least one pair (i, J)

$$\frac{d(\alpha^{iA}x^{iA} + \alpha^{iB}x^{iB})}{dp^J} > 0 \qquad \text{in } U$$

(i.e. at least in one country i total demand $\hat{x}^{iA} + \hat{x}^{iB}$ depends *positively* on the price of one of the product varieties).

Proof: see appendix.

The idea of the proof is the following. Consider e.g. a point p in U that is close to the reaction curve R^{1A}. In this point, π_A^{1A} is close to zero, and the weight α^{1A} must be large relative to α^{2A}. A sufficiently high weight α^{1A} renders the derivative of country 1's total demand $\frac{d(\hat{x}^{1A} + \hat{x}^{1B})}{dp^B} = \alpha^{1A}x_B^{1A} + \alpha^{1B}x_B^{1B}$ positive, as $x_B^{1A} > 0$. It is true that the effect of α^{1A} can be offset by choosing a high α^{1B} while leaving the ratio α^{1B}/α^{2B} constant, but it is shown in the proof that thereby the problem is shifted to country 2.

While it is not a priori impossible that total demand increases when one of the prices rises[3], it is certainly an exception. Proposition 1 implies that, if we make the common assumption that total demand depends negatively on both prices, not all points $p \in U$ can be generated as a uniform price equilibrium by choosing appropriate weights on the demand functions any more. In particular, it is not clear whether in case of best response asymmetry, it is indeed possible that price discrimination leads to an increase (or a decrease) of all prices.

[3] This may occur, if the product is vertically differentiated and quality can be substituted for by quantity, which is e.g. the case, if quality includes durability. If the more durable product variety becomes more expensive, total demand will rise.

4 Identical structure of both countries

This section uses additional symmetry assumptions to analyse the question whether in case of best-response asymmetry all prices can move in the same direction if price discrimination is introduced. The countries are assumed to be mirror images of each other in the sense that demand for the domestic product variety (for the imported product variety) is described by the same function, i.e.

$$x^{1A}(p', p'') = x^{2B}(p'', p') \;\text{ and }\; x^{1B}(p', p'') = x^{2A}(p'', p') \;\text{ for all prices }\; p',\, p''. \qquad (6)$$

In particular, corresponding derivatives of the demand and profit functions are equal, e.g. $x^{1A}_B = x^{2B}_A$ and $\pi^{2A}_{AA} = \pi^{1B}_{BB}$, and all parameters can be expressed in terms of country 1. Moreover, the equilibria will be symmetric, i.e. if markets are segmented and price discrimination is possible, we have $p^{1A}_{seg} = p^{2B}_{seg}$ and $p^{1B}_{seg} = p^{2A}_{seg}$, and the uniform prices of both firms will be equal, $p^A_u = p^B_u$.

Proposition 2 Assume best-response asymmetry holds for the profit functions π^{iJ}, and the countries are symmetric in the sense of mirror images. Define the region $A = U \cap \{(p^A, p^B) |\, p^A > p^B\}$.

(i) If $\pi^{1A}_{AA} + \pi^{1B}_{BA} < 0$ and $\pi^{1B}_{BB} + \pi^{1A}_{AB} < 0$ for all $(p^{1A}, p^{1B}) \in A$,
then $p^{1A}_{seg} = p^{2B}_{seg} > p^A_u = p^B_u > p^{1B}_{seg} = p^{2A}_{seg}$,
i.e. the equilibrium uniform price lies between the discriminating prices.

(ii) If $\pi^{1A}_{AA} + \pi^{1B}_{BA} > 0$ for all $(p^{1A}, p^{1B}) \in A$,
$p^{1A}_{seg} = p^{2B}_{seg} > p^{1B}_{seg} = p^{2A}_{seg} > p^A_u = p^B_u$,
i.e. the uniform price is lower than all prices in the discriminating equilibrium.

(iii) If $\pi^{1B}_{BB} + \pi^{1A}_{AB} > 0$ for all $(p^{1A}, p^{1B}) \in A$,
then $p^A_u = p^B_u > p^{1A}_{seg} = p^{2B}_{seg} > p^{1B}_{seg} = p^{2A}_{seg}$,
i.e. the uniform price is higher than all prices in the discriminating equilibrium.

Proof: Due to the additional symmetry assumptions, the proposition can be proved using a continuous approach. Let s be the maximum feasible price difference for each firm, i.e. the conditions $p^{1A} - p^{2A} \leq s$ and $p^{2B} - p^{1B} \leq s$ must be satisfied[4]. If the constraint is binding, firms can actually set only one price, and profits equal

$$\pi^A = \pi^{1A} + \pi^{2A} = (p^{1A} - c)x^{1A} + (p^{1A} - s - c)x^{2A} \tag{7}$$

$$\pi^B = \pi^{1B} + \pi^{2B} = (p^{2B} - s - c)x^{1B} + (p^{2B} - c)x^{2B}. \tag{8}$$

If the maximum price difference s changes, firms adjust their prices according to

$$\frac{dp^{1A}}{ds} = \frac{dp^{2B}}{ds} = \frac{\pi^{1B}_{BB} + \pi^{1A}_{AB}}{\pi^{1A}_{AA} + \pi^{1A}_{AB} + \pi^{1B}_{BB} + \pi^{1B}_{BA}} \tag{9}$$

$$\frac{dp^{1B}}{ds} = \frac{dp^{2A}}{ds} = -\frac{\pi^{1A}_{AA} + \pi^{1B}_{BA}}{\pi^{1A}_{AA} + \pi^{1A}_{AB} + \pi^{1B}_{BB} + \pi^{1B}_{BA}}. \tag{10}$$

The common denominator $\pi^{1A}_{AA} + \pi^{1A}_{AB} + \pi^{1B}_{BB} + \pi^{1B}_{BA}$ is negative by the standard assumptions (1)-(3). Thus the sign of the expressions depends on the signs of the numerators.

We continue to prove case (ii) of the proposition; the other two cases can be proved analogously. By assumption $\pi^{1A}_{AA} + \pi^{1B}_{BA} > 0$ for all $(p^{1A}, p^{1B}) \in A$, hence $\frac{dp^{1B}}{ds} = \frac{dp^{2A}}{ds} > 0$. This means that a decreasing maximum price difference ($ds < 0$) induces the lower export prices to fall, meaning that for $s = 0$, the uniform prices $p^A_u = p^B_u$ are lower than the export prices in case of segmented markets, $p^{1B}_{seg} = p^{2A}_{seg}$. As the domestic prices in case of price discrimination, $p^{1A}_{seg} = p^{2B}_{seg}$ are higher than the export prices $p^{1B}_{seg} = p^{2A}_{seg}$, this is sufficient to prove (ii) of the proposition. ■

The conditions of (i), (ii) and (iii) of the proposition exclude each other, but they are not exhaustive, as with general demand functions, it is possible that each of the three conditions holds in some part of A. Thus the proposition gives sufficient, but not necessary conditions. Moreover, the argument of the proposition cannot be applied to asymmetric demand functions. It remains of course possible to determine expressions for $\frac{dp^{iJ}}{ds}$, but these cannot directly serve to compare the uniform prices and the discriminating prices, as the range of s in which the maximum price difference is a binding constraint is different for the two firms.

[4] The parameter s can be interpreted as the cost of arbitrage between the two countries or markets.

If demand functions are linear, it follows directly that case (i) of the proposition applies.

Corollary Assume that best-response asymmetry holds and that in each country, total demand depends negatively on both prices (i.e. $x_J^{1A} + x_J^{1B} < 0$ and $x_J^{2A} + x_J^{2B} < 0$, $J = A, B$).

If demand functions are linear and the two countries are symmetric in the sense of mirror images, each firm lowers its high price and increases its low price if price discrimination is not possible any more.

Proof:

$$\begin{aligned} \pi_{AA}^{1A} + \pi_{BA}^{1B} &= 2x_A^{1A} + (p^{1A} - c)x_{AA}^{1A} + x_A^{1B} + (p^{1B} - c)x_{BA}^{1B} \\ & \quad 2x_A^{1A} + x_A^{1B} < x_A^{1A} < 0, \end{aligned} \tag{11}$$

$$\begin{aligned} \pi_{BB}^{1B} + \pi_{AB}^{1A} &= 2x_B^{1B} + (p^{1B} - c)x_{BB}^{1B} + x_B^{1A} + (p^{1A} - c)x_{AB}^{1A} \\ & \quad 2x_B^{1B} + x_B^{1A} < x_B^{1B} < 0, \end{aligned} \tag{12}$$

thus case (i) of proposition 2 applies. ■

Note that the inequalities 11 and 12 hold with some tolerance, thus they continue to hold for some curvature of the demand functions. In this sense, the result that firms choose intermediate prices when price discrimination is not possible any more can be regarded as the "normal case". Exceptions, that in the case of best-response asymmetry may occur, are either based on asymmetry or on non-linearity of the demand functions. In the following, these two possibilities are considered in turn. In the next section, an asymmetry is introduced into a model with linear demand, whereas the numerical example in section 6 involves symmetric but non-linear demand systems.

5 Linear Demand Functions

In this section it is shown that with linear demand functions it is possible that one firm increases (decreases) both of its prices in response to introducing price discrimination, resulting in prices rising (falling) unambiguously in one country, while in the other country the domestic price increases and the import price falls. It is, however, not possible that all prices move in the same direction.

Proposition 3 Assume that in each country, total demand depends negatively on both prices. If the demand functions $x^{iJ},\ i = 1, 2,\ J = A, B$, are linear

(i) it is possible that three of the four prices rise in response to the introduction of price discrimination, and it is also possible that three of the four prices rise,

(ii) but it is not possible that all prices move in the same direction.

Example I	Example II
$x^{1A} = 78 - 8p^{1A} + 4p^{1B}$ $x^{1B} = 39,5 - 5p^{1B} + 4p^{1A}$ $x^{2A} = 24 - 5p^{2A} + 4p^{2B}$ $x^{2B} = 72 - 8p^{2B} + 4p^{2A}$	$x^{1A} = 72 - 5p^{1A} + 4p^{1B}$ $x^{1B} = 96 - 8p^{1B} + 4p^{1A}$ $x^{2A} = 24 - 8p^{2A} + 4p^{2B}$ $x^{2B} = 72 - 5p^{2B} + 4p^{2A}$
$p^{1A}_{seg} = 6,29 \quad p^{2A}_{seg} = 4,67 \quad p^{A}_{u} = 5,69$ $p^{1B}_{seg} = p^{2B}_{seg} = 5,67 \quad p^{B}_{u} = 5,73$	$p^{1A}_{seg} = 10,67 \quad p^{2A}_{seg} = 3,67 \quad p^{A}_{u} = 6,27$ $p^{1B}_{seg} = p^{2B}_{seg} = 8,67 \quad p^{B}_{u} = 8,39$

Table 2: Examples showing proposition 3(i)

Proof: (i) see examples in table 5.
(ii) see appendix.

Part (i) of the proposition can be proved by two examples, which are given in table 5. The examples are constructed by assuming best-response asymmetry. The countries are symmetric in the sense of mirror images with the exception of the constant terms. Moreover, the special case is considered that in the segmented market equilibrium, firm B sets the same price in both countries, i.e. $p^{1B}_{seg} = p^{2B}_{seg}$, whereas firm A sets a higher price on its domestic market, i.e. $p^{1A}_{seg} > p^{2A}_{seg}$ (see figure 2). Although with linear demand curves, it can be solved for all prices explicitly, a direct comparison of the prices to prove part (ii) of the proposition exceeds tractability. The proof, that is given in the appendix, is based on a continuous approach increasing the maximal feasible price difference s.

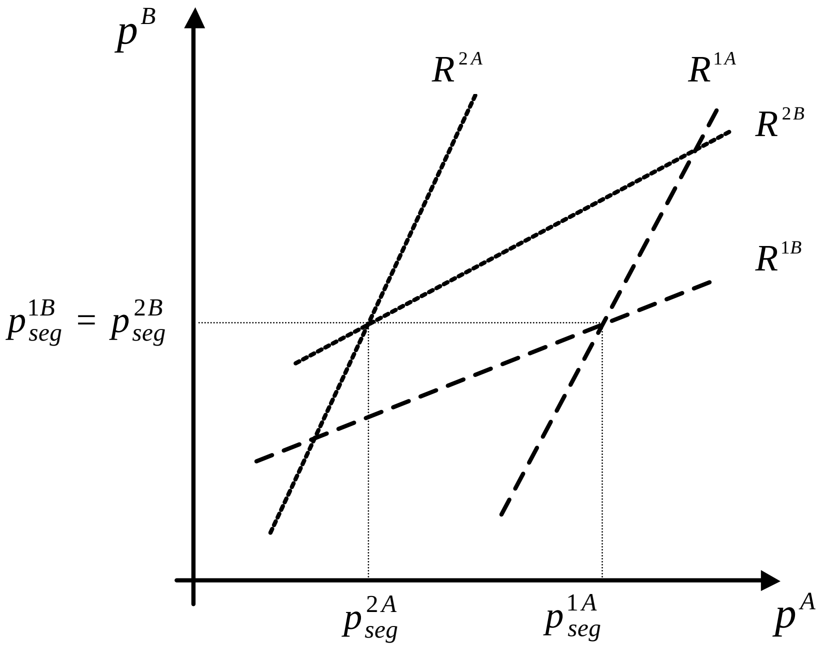

Figure 2: Special case of best-response asymmetry

6 A Numerical Example

In this section, we give a numerical example of symmetric demand functions that lead to a uniform price equilibrium with prices that are unequivocally higher than all prices in the discriminating equilibrium. Assume that total demand in each country is constant and normalized to 1. Demand for the different product varieties depends on the relative price. In particular, the demand function for firm A in its home country 1 is

$$x^{1A} = \frac{1}{1 + (\frac{p^{1A}}{p^{1B}\phi})^{\gamma}}, \qquad \phi > 1,\ \gamma > 2, \tag{13}$$

and on its export market demand equals

$$x^{2A} = \frac{1}{1 + (\frac{p^{2A}\phi}{p^{2B}})^{\gamma}}. \tag{14}$$

The demand function for firm B are mirror images of these functions as described in section 4. The parameter $\phi > 1$ represents a home bias in demand: For the respective foreign firm, it "inflates" the relative price, and for the home firm, it "deflates" it. Marginal production costs equal $c = 1$. For the parameters $\gamma = 4$ and $\phi = 1.2$, numerical computation gives the following result. If markets are segmented, the equilibrium price on the domestic markets is $p^{1A} = p^{2B} = 2,17$ and the price on the export market will be $p^{2A} = p^{1B} = 1,87$. If no price discrimination is possible, equilibrium uniform prices will be $p^{1A} = p^{2B} = p^{2A} = p^{1B} = 2,32$, which is higher than all prices in the discriminating equilibrium.

7 Summary and Discussion

The present paper analyses price discrimination in oligopolies with best-response asymmetry, i.e. the strong market of one firm is the weak market of the other. The focus is on the possibility that all prices move in the same direction when price discrimination is introduced — a result that cannot occur in case of a monopolist who will always raise the price on one submarket and lower it on the other. It is shown that equilibrium uniform prices lie between the discriminating prices in oligopolies with best-response asymmetry, too, if the demand functions do not deviate too much from linearity and symmetry (in the sense of mirror images of the submarkets). However, differing demand structures of the consumer groups or sufficiently large non-linearities may cause a firm's uniform price to be higher or lower than both of its discriminating prices.

By introducing an asymmetry of the parameters into a model with linear demand functions, we show that it is possible that one of the two firms increases the price of its product variety on both submarkets as well as that it decreases its price on both submarkets when price discrimination is introduced. This means that three of the four prices move in the same direction.

Moreover, we give a numerical example with non-linear demand functions in which the uniform prices are higher than all prices in the discriminating equilibrium. The intuition of the latter case is that if price discrimination is not possible, each firm will focus on its more attractive "home market", thus competition will be weakened. Price discrimination does not only give firms the possibility to exploit varying price elasticities on different markets. In case of an oligopoly, it also creates an additional dimension of competition. It is well-known that a monopolist cannot loose by having additional possibilities, as he need not use them. In contrast, when there is strategic interaction between firms, all firms may be worse off in the new equilibrium (Bester and Petrakis 1996, Corts 1998).

There is no corresponding interpretation of the opposite case that the uniform price is lower than all prices in case of price discrimination (and to the best of our knowledge, no example — neither numerical nor empirical — is discussed in the literature), suggesting the conjecture, that this case can be excluded by rather weak assumptions. However, the formal considerations in section 4 and 5 qualify the argument, as the question whether the uniform price can be higher or lower than all prices in the discriminating equilibrium seems to be of symmetric structure. Clarifying this puzzle is ongoing research.

Appendix: Proof of Proposition 1

The proposition is shown for $\tilde{p}$ lying on the part of the boundary of U that coincides with R^{1A}. For the other segments of the boundary of U, the proof is analogous.

a) Let $\bar{U}$ be the closure of U. The derivatives x_K^{iJ} of the demand functions are continuous and different from 0, and the set $\bar{U}$ is compact. Thus, there exist positive constants $b_1, b_2 > 0$ such that for all $p \in U$,

$$b_1 < |x_K^{iJ}| < b_2 \qquad i = 1, 2, \quad J, K = A, B.$$

b) Assume that $\alpha^{1A} x_B^{1A} + \alpha^{1B} x_B^{1B} < 0$ for positive real numbers α^{1A} and α^{1B}. This implies $\frac{\alpha^{1A}}{\alpha^{1B}} < -\frac{x_B^{1B}}{x_B^{1A}} < \frac{b_2}{b_1}$. Thus $\frac{\alpha^{1A}}{\alpha^{1B}} > \frac{b_2}{b_1}$ implies that $\alpha^{1A} x_B^{1A} + \alpha^{1B} x_B^{1B} > 0$. Likewise, $\frac{\alpha^{2B}}{\alpha^{2A}} > \frac{b_2}{b_1}$ implies that $\alpha^{2B} x_A^{2B} + \alpha^{2A} x_A^{2A} > 0$.

c) As the derivative of the profit function π_B^{2B} is continuous, it is bounded on $\bar{U}$, thus there exists $b_3 > 0$ such that $|\pi_B^{2B}| < b_3$. Moreover, there exists $\varepsilon_1 > 0$ and $b_4 > 0$ such that in the ε_1-neighborhood $V_{\varepsilon_1}(\tilde{p})$ $\tilde{p}$ of we have $|\pi_A^{2A}| > b_4$ and $|\pi_B^{1B}| > b_4$, as $\tilde{p}$ has a positive distance both from R^{2A} and from R^{1B}.

d) Let $a = \frac{b_1^2\, b_4^2}{b_2^2\, b_3}$. As $\pi_A^{1A}(\tilde{p}) = 0$, there exists $\varepsilon_2 > 0$ such that $|\pi_A^{1A}| < a$ in the ε_2-neighborhood $V_{\varepsilon_2}(\tilde{p})$ of $\tilde{p}$.

e) Let $V(\tilde{p}) = V_{\varepsilon_1}(\tilde{p}) \cap V_{\varepsilon_2}(\tilde{p})$. Moreover, according to the assumption of the proposition, let $p \in V(\tilde{p}) \cap U$ and

$$\alpha^{1A} \pi_A^{1A}(p) + \alpha^{2A} \pi_A^{2A}(p) = 0 \quad \text{and} \quad \alpha^{1B} \pi_B^{1B}(p) + \alpha^{2B} \pi_B^{2B}(p) = 0.$$

Using step c) and d), it follows that

$$\frac{\alpha^{1A}}{\alpha^{2A}} = \frac{|\pi_A^{2A}(p)|}{|\pi_A^{1A}(p)|} > \frac{b_4}{a} \quad \text{and} \quad \frac{\alpha^{2B}}{\alpha^{1B}} = \frac{|\pi_B^{1B}(p)|}{|\pi_B^{2B}(p)|} > \frac{b_4}{b_3},$$

and thus

$$\frac{\alpha^{1A}}{\alpha^{1B}} \frac{\alpha^{2B}}{\alpha^{2A}} = \frac{\alpha^{1A}}{\alpha^{2A}} \frac{\alpha^{2B}}{\alpha^{1B}} > \frac{b_4}{a} \frac{b_4}{b_3} = \left(\frac{b_2}{b_1}\right)^2.$$

Hence, $\frac{\alpha^{1A}}{\alpha^{1B}} > \frac{b_2}{b_1}$ or $\frac{\alpha^{2B}}{\alpha^{2A}} > \frac{b_2}{b_1}$, which by step b) proves the proposition.

Proof the Proposition 3(ii):

We show that it is not possible that all prices fall, when price discrimination is introduced, i.e. that at least one of the four terms $p_u^J - p_{seg}^{iJ}$, $J = A, B$, $i = 1, 2$, is positive. The conjecture that at least one of these four differences is positive can be proved analogously.

For the proof, a continuous approach of the changes in prices is applied. Let s denote the maximum feasible price difference. In steps a) to c) of the proof, it is analysed how prices change locally when s changes marginally. Steps d)to f) considers the total price changes $p_u^J - p_{seg}^{iJ}$.

a) Assume that the condition that $|p^{1J} - p^{2J}| \leq s$ is binding for both firms, and that the firms set a higher price on different markets, w.l.o.g. $p^{1A} > p^{2A}$ and $p^{2B} > p^{1B}$, i.e. $p^{1A} - s = p^{2A}$ and $p^{2B} - s = p^{1B}$. In this case, the first order conditions for the profit maximization of the two firms read

$$\pi_A^{1A}(p^{1A}, p^{2B} - s) + \pi_A^{2A}(p^{1A} - s, p^{2B}) = 0$$

$$\pi_B^{1B}(p^{1A}, p^{2B} - s) + \pi_B^{2B}(p^{1A} - s, p^{2B}) = 0$$

If s changes, equilibrium prices change according to

$$\left.\frac{dp^{1A}}{ds}\right|_{both} = \frac{1}{D_1}\left[\left(2x_A^{2A} + x_B^{1A}\right)\left(2x_B^{1B} + 2x_B^{2B}\right) - \left(2x_B^{1B} + x_A^{2B}\right)\left(x_B^{1A} + x_B^{2A}\right)\right] \gtreqless 0$$
$$\left.\frac{dp^{2B}}{ds}\right|_{both} = \frac{1}{D_1}\left[-\left(2x_A^{2A} + x_B^{1A}\right)\left(x_A^{1B} + x_A^{2B}\right) + \left(2x_B^{1B} + x_A^{2B}\right)\left(2x_A^{1A} + 2x_A^{2A}\right)\right] \gtreqless 0$$

where $D_1 = (\pi_{AA}^{1A} + \pi_{AA}^{2A})(\pi_{BB}^{1B} + \pi_{BB}^{2B}) - (\pi_{AB}^{1A} + \pi_{AB}^{2A})(\pi_{BA}^{1B} + \pi_{BA}^{2B}) > 0$.

The sign of these expressions is ambiguous, but it can be shown that at most one of the two multipliers can be negative. Note that $\left(2x_A^{2A} + x_B^{1A}\right) + \left(2x_B^{1B} + x_A^{2B}\right) = 2x_A^{2A} + x_A^{2B} + 2x_B^{1B} + x_B^{1A} < 0$ by assumption. Thus at most one of the terms $\left(2x_A^{2A} + x_B^{1A}\right)$ and $\left(2x_B^{1B} + x_A^{2B}\right)$ can be positive. Assume that $\left(2x_A^{2A} + x_B^{1A}\right) > 0$, implying that

$\left(2x_B^{1B} + x_A^{2B}\right) < 0$ and that $\left|2x_B^{1B} + x_A^{2B}\right| > \left|2x_A^{2A} + x_B^{1A}\right|$. It follows that $\left.\frac{dp^{2B}}{ds}\right|_{both} > 0$, as for the numerator the following holds

$$
\begin{aligned}
& -\left(2x_A^{2A} + x_B^{1A}\right)\left(x_A^{1B} + x_A^{2B}\right) + \left(2x_B^{1B} + x_A^{2B}\right)\left(2x_A^{1A} + 2x_A^{2A}\right) \\
> \; & \left(2x_B^{1B} + x_A^{2B}\right)\left(x_A^{1B} + x_A^{2B}\right) + \left(2x_B^{1B} + x_A^{2B}\right)\left(2x_A^{1A} + 2x_A^{2A}\right) \\
= \; & \left(2x_B^{1B} + x_A^{2B}\right)\left(x_A^{1B} + x_A^{2B} + 2x_A^{1A} + 2x_A^{2A}\right) > 0.
\end{aligned}
$$

Analogously, $2x_B^{1B} + x_A^{2B} > 0$ implies that $\frac{dp^{1A}}{ds} > 0$.

b) Assume that the condition that $|p^{1J} - p^{2J}| \leq s$ is binding for both firms, and that both firms set their higher price on the same market. W.l.o.g. $p^{1A} > p^{2A}$ and $p^{1B} > p^{2B}$, i.e. $p^{1A} - s = p^{2A}$ and $p^{1B} - s = p^{2B}$. The first order conditions for the profit maximization of the two firms are

$$\pi_A^{1A}(p^{1A}, p^{1B}) + \pi_A^{2A}(p^{1A} - s, p^{1B} - s) = 0$$

$$\pi_B^{1B}(p^{1A}, p^{1B}) + \pi_B^{2B}(p^{1A} - s, p^{1B} - s) = 0,$$

implying [5]

$$
\begin{aligned}
\left.\frac{dp^{1A}}{ds}\right|_{bind} &= \frac{1}{D_2}[(\pi_{AA}^{2A} + \pi_{AB}^{2A})(\pi_{BB}^{1B} + \pi_{BB}^{2B}) - (\pi_{BB}^{2B} + \pi_{BA}^{2B})(\pi_{AB}^{1A} + \pi_{AB}^{2A})] > 0 \\
\left.\frac{dp^{1B}}{ds}\right|_{bind} &= \frac{1}{D_2}[-(\pi_{AA}^{2A} + \pi_{AB}^{2A})(\pi_{BA}^{1B} + \pi_{BA}^{2B}) + (\pi_{AA}^{1A} + \pi_{AA}^{2A})(\pi_{BB}^{2B} + \pi_{BA}^{2B})] > 0,
\end{aligned}
$$

where $D_2 = (\pi_{AA}^{1A} + \pi_{AA}^{2A})(\pi_{BB}^{1B} + \pi_{BB}^{2B}) - (\pi_{AB}^{1A} + \pi_{AB}^{2A})(\pi_{BA}^{1B} + \pi_{BA}^{2B}) > 0$.

---

[5]The sign of these expressions is unambiguous not only for linear demand functions.

c) Assume that the condition that $|p^{1J} - p^{2J}| \leq s$ is binding only for firm A. Firm B can choose its prices for the two countries separately, thus it maximizes its profits π^{1B} and π^{2B} independently. In contrast, firm A is restricted by the condition $p^{2A} = p^{1A} - s$. The first order conditions for profit maximization are

$$\pi_A^{1A}(p^{1A}, p^{1B}) + \pi_A^{2A}(p^{1A} - s, p^{2B}) = 0$$

$$\pi_B^{1B}(p^{1A}, p^{1B}) = 0$$

$$\pi_B^{2B}(p^{1A} - s, p^{2B}) = 0.$$

If s rises marginally, prices change according to

$$\left.\frac{dp^{1A}}{ds}\right|_{nonb} = \frac{2x_{BB}^{1B}}{\Delta}\left(4x_{AA}^{2A} \cdot x_{BB}^{2B} - x_{AB}^{2A} \cdot x_{BA}^{2B}\right) > 0$$

$$\left.\frac{dp^{1B}}{ds}\right|_{nonb} = -\frac{x_{BA}^{1B}}{\Delta}\left(4x_{AA}^{2A} \cdot x_{BB}^{2B} - x_{AB}^{2A} \cdot x_{BA}^{2B}\right) > 0$$

$$\left.\frac{dp^{2B}}{ds}\right|_{nonb} = \frac{x_{BA}^{2B}}{\Delta}\left(4x_{AA}^{1A} \cdot x_{BB}^{1B} - x_{AB}^{1A} \cdot x_{BA}^{1B}\right) < 0$$

$$\left.\frac{dp^{2A}}{ds}\right|_{nonb} = -\frac{2x_{BB}^{2B}}{\Delta}\left(4x_{AA}^{1A} \cdot x_{BB}^{1B} - x_{AB}^{1A} \cdot x_{BA}^{1B}\right) < 0$$

where $\Delta = \pi_{BB}^{2B}\left(\pi_{AA}^{1A} \cdot \pi_{BB}^{1B} - \pi_{AB}^{1A} \cdot \pi_{BA}^{1B}\right) + \pi_{BB}^{1B}\left(\pi_{AA}^{2A} \cdot \pi_{BB}^{2B} - \pi_{AB}^{2A} \cdot \pi_{BA}^{2B}\right) < 0$. Hence p^{1A} and p^{1B} rise, whereas p^{2A} and p^{2B} fall, when the maximum feasible price difference rises.

d) W.l.o.g., $p_{seg}^{1A} \geq p_{seg}^{2A}$ and $p_{seg}^{1A} - p_{seg}^{2A} \geq |p_{seg}^{2B} - p_{seg}^{1B}| \geq 0$. Define $\overline{s} = p_{seg}^{1A} - p_{seg}^{2A}$. Then there exists $0 \leq \underline{s} \leq \overline{s}$, such that the restriction $|p^{1J} - p^{2J}| \leq s$ is binding only for firm A as long as $\underline{s} < s \leq \overline{s}$, whereas for $0 \leq s \leq \underline{s}$, both firms are restricted by the maximum price difference s. To finish the proof, we have to distinguish the cases $p_{seg}^{2B} - p_{seg}^{1B}| \geq 0$ (step e) and $p_{seg}^{2B} - p_{seg}^{1B}| \geq 0$ (step f).

e) Assume $p^{1A}_{seg} \geq p^{2A}_{seg}$ and $p^{2B}_{seg} \geq p^{1B}_{seg}$

The differences between the uniform prices and the discriminating prices can be expressed as

$$
\begin{aligned}
p^A_u - p^{1A}_{seg} &= \underline{s} \left. \frac{dp^{1A}}{ds} \right|_{both} + (\overline{s} - \underline{s}) \left. \frac{dp^{1A}}{ds} \right|_{nonb} \\
p^B_u - p^{2B}_{seg} &= \underline{s} \left. \frac{dp^{2B}}{ds} \right|_{both} + (\overline{s} - \underline{s}) \left. \frac{dp^{2B}}{ds} \right|_{nonb} .
\end{aligned}
$$

By step a) and b)if $\left. \frac{dp^{1A}}{ds} \right|_{both} > 0$, it follws that $p^A_u - p^{1A}_{seg} > 0$, which proves the conjecture in this case. Thus assume that $\left. \frac{dp^{1A}}{ds} \right|_{both} < 0$, which by step a) implies that $\left. \frac{dp^{2B}}{ds} \right|_{both} > 0$. To prove that in this case, $p^A_u - p^{1A}_{seg} > 0$ or $p^B_u - p^{2B}_{seg} > 0$, it suffices to show that

$$
\Gamma = \left. \frac{dp^{1A}}{ds} \right|_{nonb} \left. \frac{dp^{2B}}{ds} \right|_{both} - \left. \frac{dp^{1A}}{ds} \right|_{both} \left. \frac{dp^{2B}}{ds} \right|_{nonb} > 0.
$$

This is the case as it can be shown (by a simple but tedious computation) that

$$
\begin{aligned}
\Gamma \;\; = \;\; & -\frac{1}{\Delta D_1} \left(4x^{1B}_B x^{2A}_A - x^{1A}_B x^{2B}_A\right) \left[(-2x^{2B}_B + x^{2B}_A\right) \left(4x^{1A}_A x^{1B}_B - x^{1A}_B x^{1B}_A\right) \\
& +(-2x^{1B}_B + x^{1B}_A)(4x^{2A}_A x^{2B}_B - x^{2A}_B x^{2B}_A)] > 0.
\end{aligned}
$$

f) Assume $p^{1A}_{seg} \geq p^{2A}_{seg}$ and $p^{2B}_{seg} \leq p^{1B}_{seg}$

In this case by step b) and c) we have

$$
p^A_u - p^{1A}_{seg} = \underline{s} \left. \frac{dp^{1A}}{ds} \right|_{bind} + (\overline{s} - \underline{s}) \left. \frac{dp^{1A}}{ds} \right|_{nonb} > 0,
$$

which finishes the proof of the theorem.

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